MATH 7A - FINAL EXAM
SAMPLE

NAME: Solns

SHOW ALL WORK NEATLY AND CLEARLY MARK YOUR ANSWERS.

This test is in two parts. On part one, you may not use a calculator; on part two, a calculator is necessary. When you complete part one, you turn it in and get part two. Once you have turned in part one, you may not go back to it.

PART ONE - NO CALCULATORS ALLOWED

Find each of the following:

(a) $\sin(60°) =$ $\sqrt{3}/2$

(b) $\csc(3\pi/4) =$ $\sqrt{2}$

(c) $\csc(225°) =$ $-\sqrt{2}$

(d) $\tan^{-1}(-\sqrt{3}) =$ $-\frac{\pi}{3}$

(e) $\cos^{-1}(\sqrt{3}/2) =$ $\pi/6$

(f) $\cos(11\pi/6) =$ $\frac{\sqrt{3}}{2}$

(g) $\sin^{-1}\pi =$ Undefined

(h) $\cot 90° =$ 0

(i) $\tan^{-1}(-1) =$ $-\frac{\pi}{4}$

(j) $\sin^{-1}(1) =$ $\frac{\pi}{2}$

(k) $\cos(150°) =$ $\frac{-\sqrt{3}}{2}$

(l) $\cot(3\pi) =$ Undefined

(m) $\sin(9\pi/4) =$ $\frac{\sqrt{2}}{2}$

(n) $\cot(3\pi/4) =$ -1

(o) $\tan(-225°) =$ -1

(p) $\sec(315°) =$ $\sqrt{2}$

(q) $\cos^2\theta + \sin^2\theta =$ 1

Note: This is actually a little longer than final

MATH7A - FINAL EXAM PART TWO
SAMPLE

NAME: Solutions

SHOW ALL WORK NEATLY. Exact answers expected unless otherwise specified. Units should be given where appropriate.

Fill in the blanks with the most appropriate answer.

(1) The slope of a line perpendicular to 3x-4y=2 is $-\frac{4}{3}$

(2) In which quadrant(s), if any, is $\cos\theta > 0$ and $\cot\theta < 0$ IV.

(3) The slant asymptote for the graph of $f(x) = \dfrac{2x^2-4x+5}{x-3}$ is $y = 2x+2$

$$\begin{array}{r|rrr} 3 & 2 & -4 & 5 \\ & & 6 & 6 \\ \hline & 2 & 2 & 11 \end{array}$$

$2x+2+\frac{11}{x-3}$

(4) $\ln e^3 =$ 3

(5) The range of f(x)= $\sin^{-1}(x)$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

(6) $\frac{\pi}{8}$ radians = 22.5 degrees

(7) $\ln\left(\sqrt[3]{\frac{xy^2}{z^3}}\right)$ can be expanded as $\frac{1}{3}\ln x + \frac{2}{3}\ln y - \ln z$

(8) Factor completely: $2x^3-54$ $2(x-3)(x^2+3x+9)$

(9) The domain of f(x)= $\cos^{-1}(x)$ is $[-1, 1]$

(10) $\log_7\left(\frac{1}{49}\right) =$ -2

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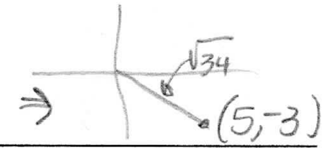
(11) Find exactly $\cos(\tan^{-1}(-3/5)) = \cos\theta = \boxed{5/\sqrt{34}}$

let $\theta = \tan^{-1}(\frac{-3}{5}) \Rightarrow \tan\theta = \frac{-3}{5}$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow$



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(12) Find the equation of the line passing through the vertex of $y=3x^2-4x+1$ and through the x-intercept of $y=\log_3(x-1)$

vertex: $x = -\frac{b}{2a} = \frac{4}{6} = \frac{2}{3}$

$y = 3(\frac{2}{3})^2 - 4(\frac{2}{3}) + 1 = -\frac{1}{3}$

$(\frac{2}{3}, -\frac{1}{3})$

x-int: $\log_3(x-1) = 0$

$3^0 = x-1$

$x = 2$

$(2,0)$

Find line through $(\frac{2}{3}, -\frac{1}{3})$ and $(2,0)$

$y = \frac{1}{4}(x-2)$

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(13) Given $f(x) = x|x|$. Rewrite f as a piecewise defined function (remove the bars).

$f(x) = x|x| = \begin{cases} x(x) & \text{if } x \geq 0 \\ x(-x) & \text{if } x < 0 \end{cases} \Rightarrow f(x) = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$

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(14) Find domain. Answer in interval notation.

(a) $f(x) = \dfrac{\log_2(x-1)}{x-5}$

Argument of log > 0 $\Rightarrow x-1>0$
$x>1$

denom. $\neq 0 \Rightarrow x-5 \neq 0$
$x \neq 5$

$\rightarrow (1,5) \cup (5,\infty)$

(b) $g(x) = \sqrt{\dfrac{x-3}{x+1}}$

radicand $\geq 0 \Rightarrow \dfrac{x-3}{x+1} \geq 0$

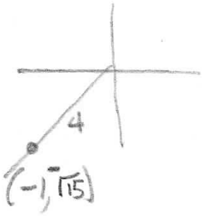
Solve with sign chart

+ − +
−1 3

$(-\infty,-1) \cup [3,\infty)$

(15) Given $\cos\theta = \frac{-1}{4}$ and θ is in Quadrant III, find:

(a) $\sin\theta =$ $-\sqrt{15}/4$ (b) $\tan\theta$ $\sqrt{15}$



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(16) Given $f(x) = 3x^2 - x$, find $\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - (x+h) - (3x^2 - x)}{h}$

$= \frac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h} = \frac{6xh + 3h^2 - h}{h} = 6x + 3h - 1$

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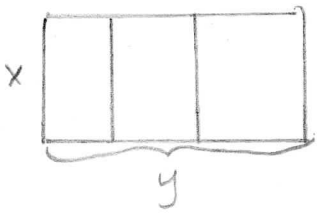
(17) A man wishes to put a fence around a rectangular field and then subdivide the field into three smaller rectangular plots by placing two fences parallel to one of the sides. If he can only afford 40 yards of fencing, what is the maximum area he can enclose?



Maximize area:

$A = xy$

need y in terms of x. $4x + 2y = 40$

$\Rightarrow y = 20 - 2x$

$A = x(20 - 2x)$

$A = 20x - 2x^2$

Max at vertex: $x = \frac{-b}{2a} = \frac{-20}{2(-2)} = 5$. When $x = 5$, Area $= 50\text{ ft}^2$

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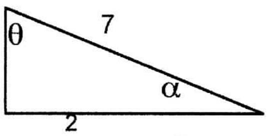
(18) Given the following right triangle, find $\cos\alpha$, , $\tan\theta$. exactly and approximate the value of θ and α

$\sqrt{45}$

$3\sqrt{5}$



$\cos\alpha =$ $\frac{2}{7}$ $\tan\theta =$ $\frac{2}{3\sqrt{5}}$ $\theta =$ 16.6° approx_, $\alpha =$ 73.4° approx .

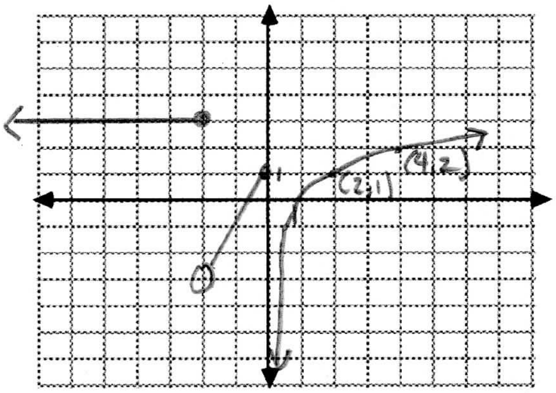
$\sin^{-1}\frac{2}{7}$ $\cos^{-1}\frac{2}{7}$

or

$90^\circ - \theta$

(19) Graph $f(x) = \begin{cases} 3 \text{ if } x \le -2 \\ 2x+1 \text{ if } -2 < x \le 0 \\ \log_2 x \text{ if } x > 0 \end{cases}$ Show scale. Label coordinates of 2 points.

(9 points)



(20) Simplify:

(a) $\dfrac{x^{-1}+y^{-2}}{x^{-2}-y^{-1}} = \dfrac{\frac{1}{x}+\frac{1}{y^2}}{\frac{1}{x^2}-\frac{1}{y}} \cdot \dfrac{x^2y^2}{x^2y^2}$

$= \dfrac{xy^2+x^2}{y^2-x^2y}$

(b) $\dfrac{x(8x-1)\left(x^2+5\right)^{-\frac{1}{2}}-8\left(x^2+5\right)^{\frac{1}{2}}}{(8x-1)^2}$

$\dfrac{(x^2+5)^{-1/2}\left[x(8x-1)-8(x^2+5)^1\right]}{(8x-1)^2}$

$\dfrac{8x^2-x-8x^2-40}{(x^2+5)^{1/2}(8x-1)^2}$

$\dfrac{-x-40}{(x^2+5)^{1/2}(8x-1)^2}$

(21) Find the equation of the line which passes through the x intercept of $f(x) = \log_3(x-1)$ and is perpendicular to the line $3x - y=8$. $\rightarrow m=3$ (15 points)

x-intercept of $y = \log_3(x-1)$ is $(2,0)$

$y-0=-\frac{1}{3}(x-3)$

$y=-\frac{1}{3}x+1$

(22) Given the polynomial $f(x) = x^4 - 4x^3 - 3x^2 + 10x + 8$,

(a) discuss end behavior $\nwarrow \nearrow$

(b) find the y intercept $(0,8)$

(c) find the x intercepts and discuss the behavior near them. Possible: $\pm\{1, 2, 4, 8\}$

(d) plot one additional point for accuracy and sketch the graph.

SHOW ALL WORK

1	1	-4	-3	10	8
		1	-3	-6	4
	1	-3	-6	4	12

$\Rightarrow (1, 12)$

2	1	-4	-3	10	8
		2	-4	-14	-8
	1	-2	-7	-4	0

$\Rightarrow (x-2)(x^3 - 2x^2 - 7x - 4)$

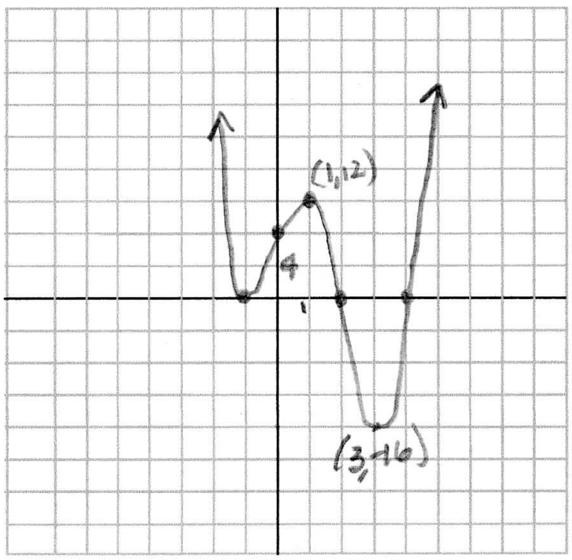
4	1	-2	-7	-4
		4	8	4
	1	2	1	0

$\Rightarrow (x-2)(x-4)(x^2+2x+1)$

$(x-2)(x-4)(x+1)^2$

X-intercepts

$(2,0)$ cross $(4,0)$ cross $(-1,0)$ turn



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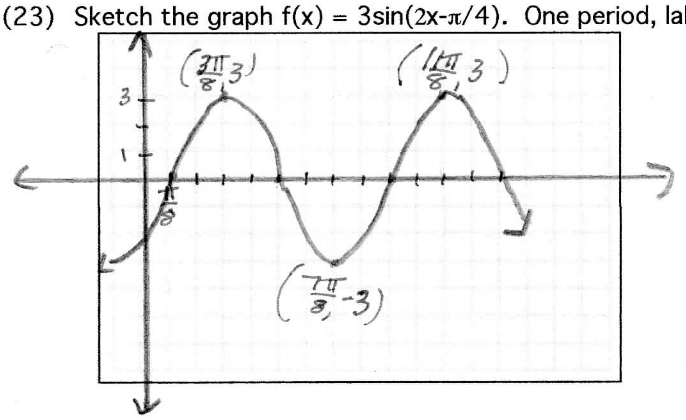
(23) Sketch the graph $f(x) = 3\sin(2x - \pi/4)$. One period, label highs and lows. (15 points)



$y = 3\sin(2(x - \frac{\pi}{8}))$

Period $= \frac{2\pi}{2} = \pi$

$\frac{1}{4}$ period $= \frac{\pi}{4}$

shift right $\frac{\pi}{8}$

(24) Sketch the graph of $y = \frac{2x^2+7x-4}{x^2+x-2} = \frac{(2x-1)(x+4)}{(x+2)(x-1)}$ (15 points)

(a) find asymptotes V.A.: $x=-2, x=1$ opposite approach H.A.: $y=2$

Cross HA?

$2 = \frac{2x^2+7x-4}{x^2+x-2}$

$2x^2+2x-4 = 2x^2+7x-4$

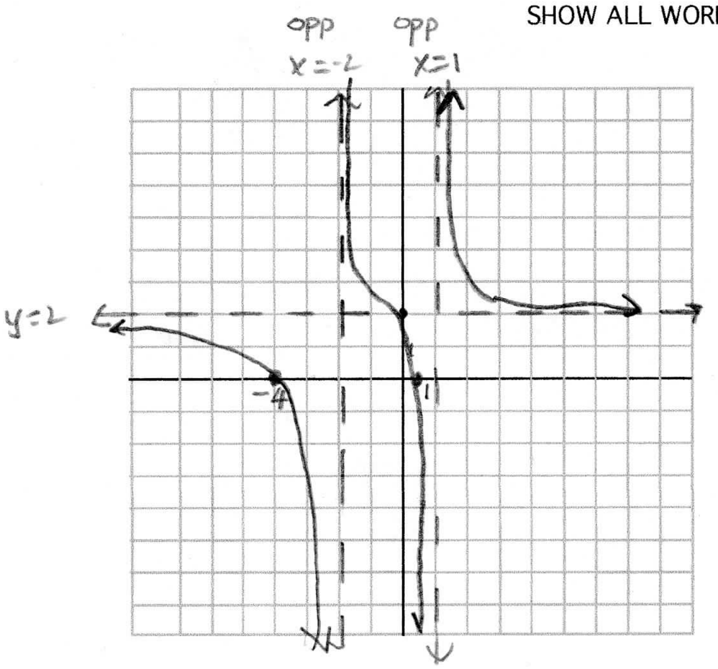
$0 = 5x$

$x = 0$

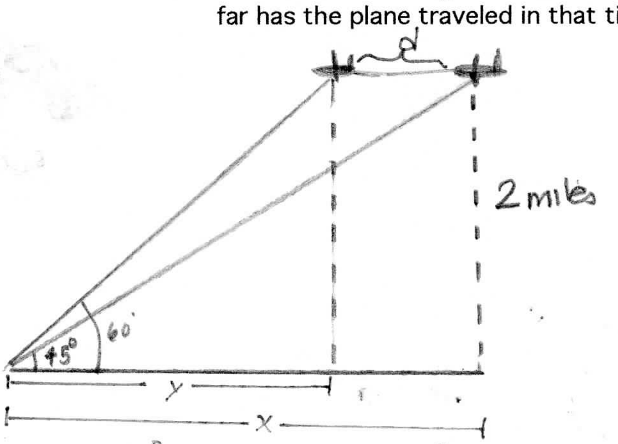
(b) find the y intercept $(0,2)$

(c) find the x intercepts and discuss the behavior near them. $(\frac{1}{2},0)(-4,0)$ Cross

SHOW ALL WORK



(25) A man looks up and sees an airplane flying in his direction at a level altitude of 2 miles. He watches the airplane for a few minutes. During that period of time he notices that the angle of elevation to the airplane changes from 45° to 60°. How far has the plane traveled in that time?



$\tan 45^\circ = \frac{2}{x}$ $\tan 60^\circ = \frac{2}{y}$

$x = \frac{2}{\tan 45^\circ}$ $y = \frac{2}{\tan 60^\circ}$

$d = x - y$

$d = \frac{2}{\tan 45^\circ} - \frac{2}{\tan 60^\circ}$

$d = 2 - \frac{2}{\sqrt{3}} \approx .85$ miles

(26) Graph $f(x) = \cot(3x)$

period $= \frac{\pi}{3}$

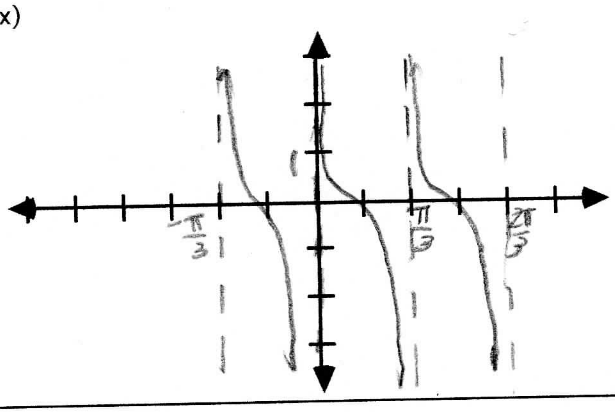
One asymptote:

$\cot(3x) = \frac{\cos(3x)}{\sin(3x)}$

$\sin 3x = 0$

$\Rightarrow x = 0$

Get others by add/subtract $\pi/3$



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(27)Solve:

(a) $6x^{-1/2} - 5x^{1/2} + x^{3/2} = 0$

$x^{-1/2}(6 - 5x + x^2) = 0$

$x^{-1/2}(3 - x)(2 - x) = 0$

never $= 0$ $\quad x = 3, 2$

(b) $(x-1)^2(2x+3) \le 0$

$-\quad -\frac{3}{2} \quad + \quad 1 \quad +$

$(-\infty, -\frac{3}{2}]$

(c) $7^{x+3} = 2^{x-3}$

$\ln 7^{x+3} = \ln 2^{x-3}$

$(x+3)\ln 7 = (x-3)\ln 2$

$x\ln 7 + 3\ln 7 = x\ln 2 - 3\ln 2$

$x\ln 7 - x\ln 2 = -3\ln 7 - 3\ln 2$

$x(\ln 7 - \ln 2) = -3\ln 7 - 3\ln 2$

$x = \frac{-3\ln 7 - 3\ln 2}{\ln 7 - \ln 2}$

(d) $\log_2(-x) = 2 - \log_2(3-x)$

$\log_2(-x) + \log_2(3-x) = 2$

$\log_2((-x)(3-x)) = 2$

$-x(3-x) = 2^2$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = \cancel{4}, -1$ $\qquad x = -1$

---

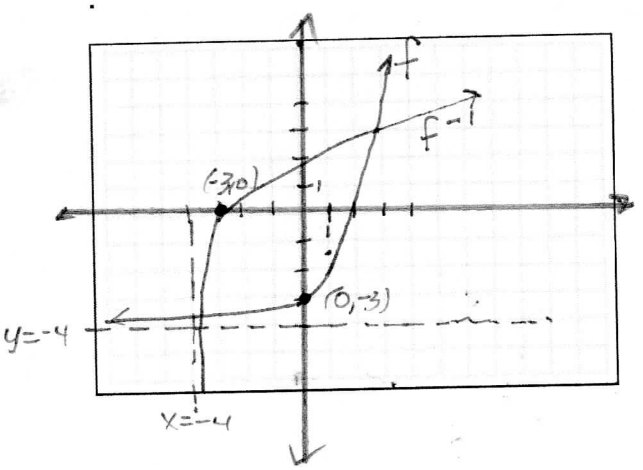
(28) Given $f(x) = e^x - 4$

(a) find $f^{-1}(x)$.

(b) Graph $f(x)$ and $f^{-1}(x)$. Label each graph and label one point on each graph.

(c) Find the domain and range for $f(x)$ and for $f^{-1}(x)$.

(15 points)



$y = e^x - 4$

switch

$x = e^y - 4$

$x + 4 = e^y$

$\ln(x+4) = \ln e^y$

$\ln(x+4) = y$

$f^{-1}(x) = \ln(x+4)$

	f	f^{-1}
domain	$(-\infty, \infty)$	$(-4, \infty)$
range	$(-4, \infty)$	$(-\infty, \infty)$